Learning by Problem-Solving

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Abstract

We present a learning by doing model that relates wages and skill development to the level of job complexity. We test our hypotheses about the job complexity effect using German Sample of Integrated Biographies data. We find that when tenure is low, wage growth is positively related to job complexity and negatively related to initial skill level, just as in our model. We calibrate the model and find that employees receive a positive wage premium to the complexity of their job and that workers in highly complex occupations acquire twice as much skills throughout life compared to less complex occupations.

JEL classification: J24, J31, D83.

1 Introduction

Since the early work of Jacob Mincer on education, experience and earnings, it became a well–known fact that wages increase with work experience at a decreasing rate. Since then, the literature in the field of human capital has accumulated vast evidence about the driving forces behind the wage curves. Wages grow with experience as a result of learning by doing (e.g. Nagypál 2007; Parsons 1972), revealing of the employee to employer match quality (Jovanovic 1979), and acquisition of firm specific (Becker 1962), industry specific (Neal 1995; Parent 2000) and occupation specific (Kambourov and Manovskii 2009) human capital.

What is less known is that wage curves vary widely across occupations. Figure 1 shows occupation–specific hourly wages for different age groups in Germany in 2006 from the Structure of Earnings Survey data. The figure suggests that occupations such as legislators, managers and professionals have wage profiles that are a level shift on the hourly rate scale, and also that the growth rates of wages of these professions are larger and remain positive longer. While ability differences could account for the level effect, they would not explain the observed patterns in the growth rates. It is also difficult to reconcile these differences with a model of uncertainty about match quality, because there is no compelling reason why match qualities would vary by occupations. Moreover,

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learning by doing models that relate productivity growth to gross output or accumulated investment require either systematic differences in the volume of activities or capital output ratios (in the case of Arrow 1962) across jobs, or sorting by individual differences to explain why learning would happen at a slower or faster rate across different occupations.

In this paper, we put forward a simple model that explains how the differences in task complexity of jobs clustered in different occupations can act as the driving force behind these earnings patterns. We then test some of the model's implications using data from administrative records of the German Social Security.

The basic idea is as follows. We view every job as a collection of tasks ("problems") of varying complexity that need to be accomplished. Every task poses a challenge and at the same time provides an opportunity to learn. Thus, jobs create a learning environment, where employees can learn by solving tasks. Just how much they can learn depends on the gap between their current skill level and the level of task complexity. More complex jobs create a better learning environment, as they provide more complex tasks and in larger quantity. Therefore, occupation–specific wage profiles are possible, because workers in different jobs accumulate skills at different rates.

We find empirical evidence that supports the model's predictions. First, there is a static effect of job complexity. Employers pay wage premia to workers employed in more complex jobs. Second, there is a dynamic effect. Workers in complex jobs develop more skills ceteris paribus.

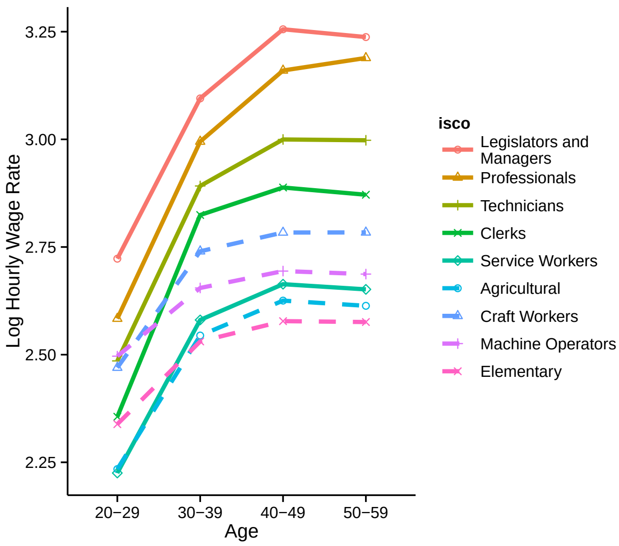

Figure 1: Earnings Profiles by Occupation.

2 The Model

In this section we analyse earnings dynamics in a single good economy. The good can be produced by a variety of jobs with labour as the only factor of production. Every job consists of a single task, and every task is uniquely characterized by its level of complexity, which we denote by a positive real number x. Higher value of x means a more complex task.

Assume that the market price of a unit of output created by completing a task of complexity x is given by a function $p(x)$, which is positive, monotone and increasing. Such dependency may result from a relation between job complexity and product quality, for example. The dependency of the market price on job complexity creates an immediate wage premium for working in a complex job. This premium is unrelated to the amount of learning or experience accumulated at the job.

Further assume that workers differ in their productivity in performing a given task, which is summarized by a task productivity function $q(x, z)$. The value of q depends on x, since the productivity of a worker on a given task changes adversely with task complexity, and on skill level z. Since we allow learning, z is a function of tenure, t, and job complexity additionally affects wages through changes in z. We write the wage equation for a worker with skill level $z(t)$ in a job with complexity x as[1]

$$\begin{aligned} &\text{wage} = W(t, x) = p(x)q(x, z(t)), \quad \text{where} \\ &\frac{dp}{dx} > 0 \qquad \frac{\partial q}{\partial x} < 0 \qquad \frac{\partial q}{\partial z} > 0. \end{aligned} \tag{1}$$

Let workers increase their skills as time progresses as a result of learning from exposure to a challenging learning environment. Since more complex jobs present a more challenging environment, workers in more complex jobs will accumulate higher skills. We suppose that the instantaneous growth rate of skills is linearly related to the gap between the level of job complexity and current level of skill:

$$\frac{\dot{z}}{z} = \alpha(\eta x - z) + \beta, \tag{2}$$

where the parameter α characterizes the rate of learning, β is a shift parameter, and η is a scale parameter.[2] In other words, the higher the cognitive distance of a worker to job requirements, the faster is the growth of skills. It is assumed

[1] The model allows a probabilistic interpretation in the following sense: working on a task can result in either a success or a failure, which is valued at 0. The likelihood of either outcome depends on a worker's skill level and on task complexity. Output $\tilde{w}$ will be a random variable with mean and variance depending on z. In one possibility, every worker is paid according to actual performance. In the other possibility, workers are paid expected wages

$$W = \mathbb{E}[\tilde{w}|z] = p(x)q(x, z),$$

where q is a bounded function modelling the probability of solving the task. In Jovanovic and Nyarko 1997, the relation is $W = 1 - \sigma_z^2$, where σ_z^2 is the variance of decision error, $0 \leq \sigma_z^2 \leq 1$. σ_z^2 can be written as $1 - q(x, z)$, thus $W = q(x, z)$ and there is no immediate job complexity premium. The dynamics of σ_z^2 is such that the wage–tenure curve is S–shaped (see Jovanovic and Nyarko 1995).

[2] We assume that α is constant. While in general it may depend on such factors as innate learning ability, curiosity, risk aversion etc., it is fixed on the level of an individual, thus we only need to worry about variability in α for empirical analysis.

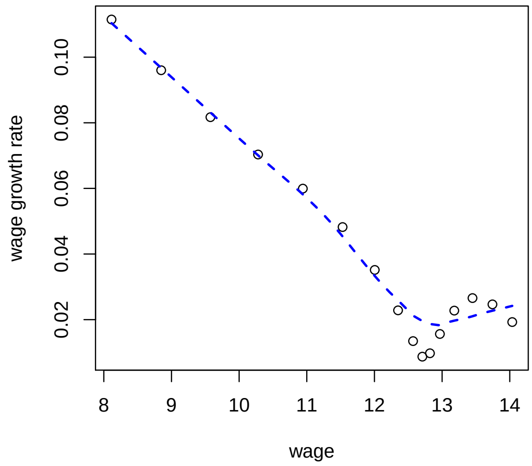

Figure 2: Relation between wage growth rate and wage.

that $\alpha > 0, \eta > 0$ and that β is such that the right hand side is not always negative.

Equation (2) may be motivated by figure 2. If we believe that wages reflect skills, then wage growth will also reflect skill growth. We approximate wage growth by a local polynomial regression of log of wage on age, and plot the estimated derivative versus the mean wage in a sample from the SIAB data, where we restrict the variation in tenure so that wages are increasing in tenure. The slope of the resulting relation is roughly linear up to high values of wages. We choose a linear relation with a negative slope to capture the essential feature of the data in an analytically tractable way. Note that the shape of the wage growth relation in figure 2 is also broadly consistent with the quadratic Mincer model.

From the Bernoulli equation (2) we obtain

$$z(t) = \left[\frac{\alpha}{\alpha\eta x + \beta} + \left(\frac{1}{z_0} - \frac{\alpha}{\alpha\eta x + \beta} \right) e^{-(\alpha\eta x + \beta)t} \right]^{-1}. \tag{3}$$

As $t \to \infty$, $z(t)$ approaches the limit $\eta x + \beta/\alpha$ irrespective of the initial value of z. When β is 0, the limiting value is x/η. For either case, we denote it by z^*. In the long run, higher job complexity leads to a higher level of skill.

We can rewrite the solution as

$$z(t) = \left[\frac{1}{z^*} + \left(\frac{1}{z_0} - \frac{1}{z^*} \right) e^{-\alpha z^* t} \right]^{-1}. \tag{4}$$

Since the time derivative of z can be written as

$$\dot{z}(t) = \alpha z(t)[z^* - z(t)], \tag{5}$$

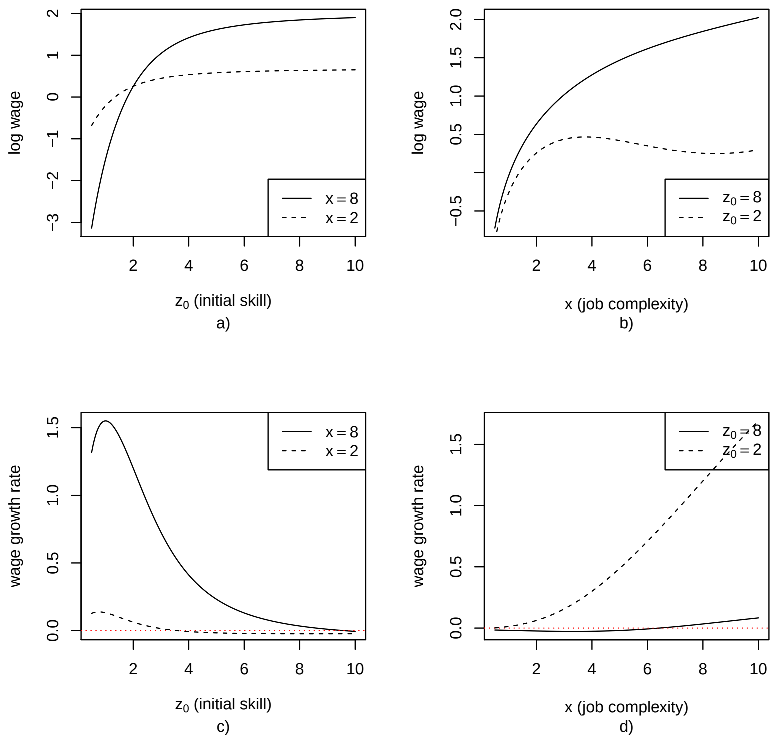

Figure 3: Relation between log wage and initial level of skill and job complexity.

it follows that z grows at a rate proportional to the distance to the limiting value:

$$\frac{\dot{z}}{z}(t) = \alpha[z^* - z(t)]. \tag{6}$$

Figure 3 shows simulated relations between the level and the rate of growth of wages and job complexity and initial level of skill implied by the model. Panel a) demonstrates that wage profiles of more complex jobs dominate for nearly all values of initial skill. Initial wages may be depressed in a highly complex job, but they will eventually catch up and exceed wages in a less complex job (not shown in the figure). In panel b), workers with relatively higher initial skill earn more in all jobs. In panel c), the rates of growth of wages in a more complex occupation dominate the rates of growth in a less complex one. In the last panel, workers with relatively lower initial skill enjoy higher rates of wage growth for the same occupation. Thus the model is able to explain such facts as higher wages and growth rates of wages in more complex occupations. Additionally, the model implies that wage growth is generally decreasing in initial skill, conditional on job complexity.

We will now analyse the properties of wages given the dynamics of z.[3] The

[3]We silently assume that $z < z^*$. Since wages are typically growing in tenure and not falling, this seems to be the common case.

elasticities of the wage rate can be expressed as

$$\varepsilon_{wx} = \frac{\partial \log W}{\partial \log x} = \epsilon_{px} + \epsilon_{qx} + \epsilon_{qz}\epsilon_{zx}, \tag{7}$$

$$\varepsilon_{wz_0} = \frac{\partial \log W}{\partial \log z_0} = \epsilon_{qz}\epsilon_{zz_0}, \tag{8}$$

$$\varepsilon_{wt} = \frac{\partial \log W}{\partial \log t} = \epsilon_{qz}\epsilon_{zt}, \tag{9}$$

where $\epsilon_{px} = x/p(x)dp(x)/dx, \epsilon_{qx} = x/q(x,z)\partial q(x,z)/\partial x$, etc. We introduce the following assumptions:

A1. $\epsilon_{px}, \epsilon_{qx}, \epsilon_{qz}$ are bounded functions of x and z.

A2. $\epsilon_{px}, \epsilon_{qx}, \epsilon_{qz}$ are constant functions of x and z.

While we mostly invoke A2, some statements can be derived with A1 alone. From

$$\epsilon_{zx} = zx\left[\frac{\eta}{z^{*2}}\left(1 - e^{-\alpha z^* t}\right) + \alpha\eta t e^{-\alpha z^* t}\left(\frac{1}{z_0} - \frac{1}{z^*}\right)\right], \tag{10}$$

$$\epsilon_{zz_0} = \frac{z}{z_0}e^{-\alpha z^* t}, \tag{11}$$

$$\epsilon_{zt} = \alpha(z^* - z)t \tag{12}$$

it follows that

$$\lim_{t\to 0}\epsilon_{zx} = 0 \qquad \lim_{t\to 0}\epsilon_{zz_0} = 1 \qquad \lim_{t\to 0}\epsilon_{zt} = 0 \tag{13}$$

$$\lim_{t\to \infty}\epsilon_{zx} = \frac{x\eta}{z^*} \qquad \lim_{t\to \infty}\epsilon_{zz_0} = 0 \qquad \lim_{t\to \infty}\epsilon_{zt} = 0 \tag{14}$$

Clearly, $\epsilon_{zx} > 0, \epsilon_{zz_0} > 0$ and $0 < \epsilon_{zt} < z^*$ for any $t > 0$. Using these properties we can find $\epsilon_{px} + \epsilon_{qx}$ and ϵ_{qz} from $\varepsilon_{wx}, \varepsilon_{wz_0}$ and ε_{wt} using assumption A2. First, we can determine ϵ_{qz}, since it is equal to $\lim_{t\to 0}\varepsilon_{wz_0}$. Then we can determine $\epsilon_{px} + \epsilon_{qx}$ from $\lim_{t\to 0}\varepsilon_{wx}$.

Table 1: Estimates of elasticities for tenure groups.

tenure	ε_{wx}	ε_{wz0}	ε_{wt}	ϵ_{zz_0}	ϵ_{zt}	ϵ_{zx}
(0,5]	0.29	0.71	0.14	1.00	0.20	0.00
(5,10]	0.35	0.76	0.22	1.07	0.31	0.06
(10,15]	0.34	0.72	0.34	1.01	0.49	0.05
(15,20]	0.36	0.64	0.53	0.91	0.75	0.07
(20,25]	0.35	0.42	0.62	0.60	0.87	0.06
(25,30]	0.35	0.67	0.93	0.95	1.32	0.06

Table 1 presents estimates of $\varepsilon_{wx}, \varepsilon_{wz_0}, \varepsilon_{wt}$, which are broadly consistent with our a priori expectations. We find that the job complexity elasticity of wage is increasing over time, just as would be predicted from ϵ_{zx} eventually reaching higher values. The elasticity with respect to z_0 is decreasing, and with respect to t increasing and reaching a peak, though we don't observe the latter elasticity eventually decreasing.[4]

We state the implication of the model about the relation between the rate of growth of wages and the model's principal variables in

Proposition 1. *The growth rate of wages is increasing in x and decreasing in z at t=0.*

Proof.

$$\lim_{t\to 0} \frac{\dot{W}}{W} = \lim_{t\to 0} \epsilon_{qz} \frac{\epsilon_{zt}}{t} = \epsilon_{qz} \left[\beta + \alpha\eta x - \alpha z_0\right]. \tag{15}$$

□

Not surprisingly, equation (15) appears to be similar to (2). Because wages reflect skills, wage growth is proportional to skill growth in the model.

3 Data, Samples and Measurement Issues

Our data come from two sources. The first source is the BIBB/IAB and BIBB/BAuA Surveys on Qualifications and Working Conditions in Germany (BIBB Surveys). The second source is the administrative records of the German Social Security System as prepared by the Institute for Employment Research (IAB).

3.1 BIBB Surveys

The BIBB Surveys are cross–sectional surveys of the German working population, which investigate job tasks, technologies, organizational and technological changes, skills and knowledge, as well as the working conditions of German employees (Rohrbach-Schmidt 2009). The survey has been repeated seven times since 1979. For the purpose of our research we are using two latest waves: 2005/2006 and 2011/2012. Each of these two waves surveys about 20,000 German employees. For each of these employees we have an extensive description of the tasks performed at the job and their qualifications including general education and months of vocational training. The BIBB Surveys are used for constructing measures of job complexity.

3.2 German Social Security Records

The IAB Employment History (BeH) contains the employment history of all Germans subject to social security since 1975 for West and since 1992 for East Germany, which covers about 80 percent of the German labour force. Civil servants and self–employed are not included in the dataset. The BeH contains information about education, occupation, industry, and work location of people

[4] This is consistent with the Mincer model results, where the tenure elasticity is first increasing, then decreasing. However, the elasticity with respect to z_0 is time invariant in the Mincer specification. One could argue that the observed pattern is due to depreciation of skills acquired at school. This would imply that the returns to schooling would be decreasing in age. We find that returns to schooling follow a U shape in the data.

on the 30th of June each year. From the available information we can calculate age, general experience, firm and occupational experience of workers. Over 350 occupations[5] are available in the data, but the IAB recommends grouping these in about 120 occupational groups to avoid false precision.

We restrict the sample to individuals who had at least seven consequent years of work experience in the same occupation. This restriction doesn't seem to bias our results, as the correlations between a dummy indicating exclusion from the final sample and our main explanatory variables are negligible. At the same time this ensures that there are no strong attrition effects in the data that may affect wage growth. From this group, we draw a random sample of individuals to be used in a cross–section regression of wage growth based on equation (15) obtained in proposition 1. We measure wage growth rate as the average of yearly growth rates of wages in the first observed year. We use occupation experience as a measure of t and years of schooling obtained from the highest qualification level as a proxy measure of $z(0)$. We assign a value of x to each individual observation based on the occupational code, thus all individuals who have the same KldB88 occupational code have the same value of x. The next section explains the specifics of obtaining our measure of job complexity x.

Due to substantial changes in returns to schooling in our data we restrict the sample to the period 1976–1990, when earnings across different education groups are relatively stable.

3.3 Job Complexity

The BIBB Surveys ask the seven questions that are arguably related to the level of job complexity. We apply principal component analysis (PCA) for extracting common variance within a set of correlated variables that we believe are strongly related to job complexity. In our case PCA is particularly appropriate not only because it can reduce the dimensionality of these questions, but also help us examine whether there is indeed a single common factor that is associated with job complexity. We conduct PCA separately for each wave. The questions and the loadings are presented in table 2. It turns out that all questions load very highly on the first component, both in 2006 and in 2012. In both analyses there is also a second component in which however the loadings are very poor. While the first component has eigenvalue of 2.35 (2.21 in 2012), the second has an eigenvalue of only 0.06 (0.10 in 2012). Therefore, the eigenvalues suggest that we should only keep the first component in each wave. Moreover, the loadings in the two independent analyses are remarkably similar. We call the scores corresponding to the first component "job complexity", because we believe that it captures the extent of problem–solving at the job.

Figure 4 compares the distributions of scores from two survey years. Both distributions are left–skewed, meaning that the mass of the workers in Germany report high incidence of new problem arrival at their jobs. The t–test of the distributions (t=-0.000) shows that their means are statistically the same.

[5]This is the 3–digit German Classification of Occupations 1988 (KldB88)

Table 2: Loadings of the first component in principal component analysis of job complexity questions in the BIBB survey.

	2006	2012
How often does it happen at your work that you		
collect, investigate and document data	.570	.571
have to react to unexpected problems and resolve these	.645	.619
have to make difficult decisions independently and without instructions	.598	.597
have to recognize and close own knowledge gaps	.546	.490
are faced with new tasks which you first have to understand and become acquainted with	.595	.585
have to improve processes or try out something new	.578	.549
have to keep an eye on many different processes at the same time	.520	.511

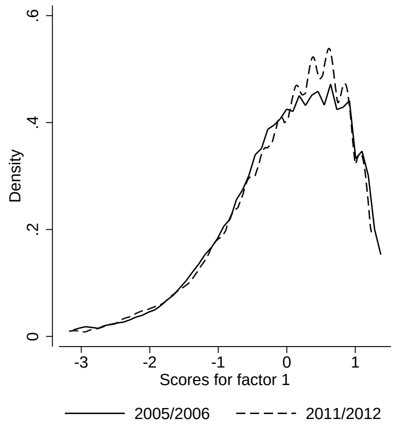

Figure 4: Distributions of job complexity factor.

4 Calibration

We calibrate the model to examine its plausibility and obtain estimates of its parameters. We use (15) to estimate α and η from the coefficients of job complexity and $z(0)$ in a cross sectional regression with wage growth as the dependent variable. Due to data limitations we have to use education as a crude proxy for $z(0)$, which will bias the coefficient of $z(0)$ upward. We find evidence of a positive effect of job complexity on wage growth and a negative effect of initial skill.

The fit is summarized in table 3. Model 1 is a baseline model which only includes year, cohort, industry, gender and Western Germany dummies. Model 2 adds years of schooling variable, which is positively related to wage growth albeit the coefficient is economically small. Model 3 estimates equation (15). We find that the coefficient on years of schooling is negative and that the coefficient on job complexity is positive. Both variables are statistically significant. We also estimate equation (15) using a different measure of $z(0)$ — mean years of schooling in the occupation that an individual has. The reason is that we

Table 3: Estimation results for wage growth equation.

	Model 1	Model 2	Model 3	Model 4
(Intercept)	0.147^{***}	0.114^{***}	0.122^{***}	0.264^{***}
	(0.000)	(0.000)	(0.000)	(0.000)
school		0.003^{***}	-0.003^{***}	
		(0.000)	(0.000)	
x			0.163^{***}	0.535^{***}
			(0.000)	(0.000)
schoolReq				-0.027^{***}
				(0.000)
R^2	0.050	0.050	0.054	0.062
Adj. R^2	0.049	0.050	0.053	0.062
Num. obs.	49519	49519	49519	49519
df	49495	49494	49493	49493
Mean dep. var	0.163	0.163	0.163	0.163
BIC	766	757	571	132
σ^2	0.243	0.243	0.243	0.242

Robust errors. Controls include gender, Western Germany, year, cohort and industry dummies.

suspect a large measurement error of the education variable in SIAB data, so we compute average years of schooling for each occupation in our sample using the BIBB survey. We see that compared to the fit in model 3, the coefficient of job complexity is larger and the coefficient on years of schooling is smaller.

Table 4: Calibrated parameters.

parameter	$\hat{\epsilon}_{qz}$	$\widehat{\epsilon_{px} + \epsilon_{qx}}$	$\hat{\alpha}$	$\hat{\eta}$	$\hat{\beta}$
estimate	0.71	0.30	0.004	54.3	0.0

In the earlier discussion we determined an estimate of $\hat{\epsilon}_{qx} = 0.7$. The regression of the rate of wage growth in table 3 yields an estimate $\hat{\alpha} = 0.004$. We get an estimate of η from the ratio of the coefficients of school and x. In order to calibrate β, we use

$$\frac{\epsilon_{zt}}{\epsilon_{zz_0}} = \alpha(z^* - z_0)t. \tag{16}$$

The graph of $\epsilon_{zt}/\epsilon_{zz_0}$ is approximately linear in tenure, and the estimated slope is $\hat{s}_t = 0.037$ (s.e. 0.003).

$$\hat{\beta} = \hat{s}_t - \hat{\alpha}(\hat{\eta}x - z_0), \tag{17}$$

We evaluate this expression at the sample average values $\bar{x} = 0.41$ and $\bar{z}_0 = 12.57$ and obtain $\hat{\beta} \approx 0$. The estimates are collected in table 4.

Figure 5 plots calibrated skill profiles for different occupations where the starting value of z_0 is taken to be the average within the corresponding occupation group. More complex occupations have higher values of starting level of skill as measured by years of schooling (the correlation coefficient of school and x in the entire SIAB sample is 0.57). In addition to that, different occupations

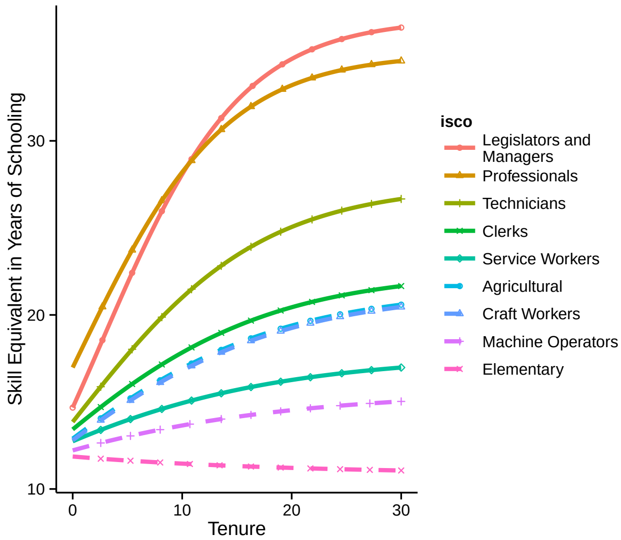

Figure 5: Calibrated skill profiles.

accrue unequal amount of skill: while professionals and technicians improve their skills by 90–100% over 30 years of tenure, clerks and craft workers by 60%, and machine operators only by 23%.

5 Discussion

We briefly discuss robustness of our findings to alternative explanations.

1. On–the–job training. In the standard mincerian earnings model, the log of wages can be written as a linear function of the starting level of training under the assumption of a linear decline of time spent in training (see Willis 1986):

$$\log W(t) = \log W(0) + \rho z_0 + \rho \left(t - \frac{t^2}{2n} \right) k(0) + \log\left(1 - k(t)\right), \quad (18)$$

   where z_0 is years of schooling, t is work experience, $k(0)$ initial fraction of time invested into training, ρ is the rate of return to schooling and n is year of retirement assumed to be fixed. Since

$$\frac{\partial^2 \log W}{\partial t \partial k(0)} = \rho \left(1 - \frac{t}{n} \right) > 0 \qquad \text{for } 0 \leq t \leq n, \quad (19)$$

   the Mincer model predicts that wage growth is increasing in starting level of training. If the amount of training is correlated with job complexity, then on–the–job training could be an important omitted variable in our specification of the wage growth derived from equation (15). We use Labor Force Survey data for privately employed Germans working at least 20 hours a week to obtain summaries on (1) the number of hours spent

Table 5: Estimation results with training.

	Model 1	Model 2	Model 3	Model 4
(Intercept)	0.270^{***}	-0.010	0.385^{***}	0.047^{*}
	(0.000)	(0.000)	(0.000)	(0.000)
x	0.087^{***}	0.347^{***}	0.532^{***}	0.372^{***}
	(0.000)	(0.000)	(0.001)	(0.001)
school	-0.006^{***}	0.005^{***}		
	(0.000)	(0.000)		
course.length	-0.002^{***}		-0.002^{***}	
	(0.000)		(0.000)	
course.attendance		-1.455^{***}		-1.300^{***}
		(0.004)		(0.008)
schoolReq			-0.030^{***}	-0.001
			(0.000)	(0.000)
R^2	0.061	0.072	0.071	0.071
Adj. R^2	0.060	0.071	0.070	0.070
Num. obs.	29486	29486	29486	29486
df	29459	29459	29459	29459
Mean dep. var	0.167	0.167	0.167	0.167
BIC	2831	2483	2512	2506
σ^2	0.253	0.251	0.251	0.251

Robust errors. Controls include gender, Western Germany, year, cohort and industry dummies.

on all taught learning activities and (2) attendance of courses, seminars, conferences, private lessons or instructions outside regular education system.[6] We find that hours spent on learning activities are uncorrelated with our measure of job complexity and negatively correlated with attendance of courses. However, the latter measure is very strongly correlated with x and years of schooling (the correlation coefficients are 0.77 and 0.71 respectively). The results of adding these variables to the model are presented in table 5. We find that training doesn't have a positive effect on wage growth. Our estimated coefficients of job complexity and years of schooling are reduced in absolute value. Years of schooling still enters the relation negatively in all estimated models except model 2.

2. Time preference. If individuals have different discount rates, they may prefer earnings profiles which generate varying streams of income. In particular, an individual i will prefer earnings profile j iff

$$\int_0^n e^{-\rho_i t} W_j(t)\, dt \geq \int_0^n e^{-\rho_i t} W_k(t)\, dt$$

for all occupation–specific wage profiles k. Thus individuals with low ρ_i may choose jobs with low starting wage and high subsequent wage growth. Let us consider wage streams that are monotone and increasing in tenure. Then it is necessary that in equilibrium occupations with higher wage growth have lower starting wage, in other words that the correlation between $W(0)$ and $\Delta W/W(0)$ is negative. We calculated the correlation

[6]Due to differences in occupational classifications, the measures are very crude.

coefficient between mean starting wage and mean wage growth after 10 years of tenure for all 126 available occupations in our data. We find that between occupations there is no negative relation between starting wage and wage growth, even conditional on differences in education.

3. Omitted ability. Since our measure of initial ability is very crude, one may argue that the results are affected by this limitation. If we assume that the measurement error of $z(0)$ is uncorrelated with x, then the bias of the coefficient of x will be downward and the bias of the $z(0)$ coefficient will be upward. Despite that, we still found a strong positive effect of x and a negative effect of $z(0)$ in table 3.

6 Conclusion

We demonstrated that the existence of occupation–specific Mincer profiles of earnings may be explained by differences in job complexity and learning induced by exposure to problem solving in more complex jobs. Jobs with higher complexity have higher static wage premia. Additionally, working in a complex jobs increases the level of skill, even after controlling for differences in education.

One empirical fact that we are not able to explain at the moment is the absence of a decline in variance of earnings with tenure. In our model, the level of skills of every worker eventually converges to the steady state value z^*. Thus, the unconditional variance of wages shrinks over time. We think that labour market information asymmetries may be responsible for much of variance dynamics that can not be explained by the model: when workers switch firms, new employers require time to learn their abilities, and this will make variance of wages grow thus possibly offsetting convergence effects.

A question that remains unanswered at this point is whether skills acquired by learning from more complex jobs are transferable to other jobs. If they are, there might be scope for designing policies that encourage workers to take challenging jobs and encourage employers to challenge their workers at the job. If these skills are not transferable, then one implication is that quit rates would be lower in more complex jobs ceteris paribus. Moreover, more work should be done to understand the matching of workers with certain abilities to jobs of certain complexity in order to understand the policy implications of possible labour market interventions.

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A Some Properties of the Model

First we will show that our model is able to generate a concave profile of wages in tenure when tenure is sufficiently large under the assumption that the initial values of z are mostly below the steady–state.

Proposition 2. *For any x, there is a t^* such that $W(t)$ is strictly concave for all $t > t^*$ and strictly convex for all $t < t^*$.*

Proof.

$$\begin{aligned}\frac{\partial^2 W}{\partial t^2} &= p(x)\left[q_{zz}\dot{z}^2 + q_z\ddot{z}\right] = p(x)\dot{z}\left[q_{zz}\dot{z} + q_z\alpha(z^* - 2z)\right]\\ &= p(x)\dot{z}\left[\frac{q_z(\epsilon_{qz}-1)}{z}\dot{z} + q_z\alpha(z^*-2z)\right]\\ &= \{\alpha p(x)\dot{z}q_z\}\left[(z^*-z)\epsilon_{qz} - z\right].\end{aligned} \tag{20}$$

The term inside left brackets is always positive and approaches 0 as $t \to \infty$. The term on the right is negative for all $t > t^*$ and positive for $t < t^*$, where

$$t^* = -\frac{1}{\alpha z^*}\log\frac{z_0}{\epsilon_{qz}(z^*-z_0)}. \tag{21}$$

Thus the wage curve has a convex segment followed by a concave one. □

B The Mincer Model

We now show how the standard Mincer regression can be obtained by an arbitrary approximation to our model. We consider z to be a function of x, z_0 and

t. We assume that p and q have continuous second order derivatives. Let the point of approximation be (x_a, z_{0a}, t_a). Then take the log of the wage equation and calculate the Taylor polynomials,

$$\log W = \log p(x) + \log q(x, z(x, z_0, t)) \tag{22}$$

$$\begin{aligned} T_1 \log W = &\log p_a + \log q_a + \left[\frac{1}{p_a}\frac{dp}{dx} + \frac{1}{q_a}\left(\frac{\partial q}{\partial x} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial x}\right)\right](x - x_a) \\ &+ \frac{1}{q_a}\frac{\partial g}{\partial z}\left[\frac{\partial z}{\partial z_0}(z_0 - z_{0a}) + \frac{\partial z}{\partial t}(t - t_a)\right], \end{aligned} \tag{23}$$

$$\begin{aligned} T_2 \log W = &T_1 \log W + \psi_x (x - x_a)^2 \\ &+ \frac{1}{2}\frac{(q_{zz}z_t^2 + q_z z_{tt})q_a - (q_z z_t)^2}{q_a^2}(t - t_a)^2 \\ &+ \frac{1}{2}\frac{(q_{zz}z_{z_0}^2 + q_z z_{z_0 z_0})q_a - (q_z z_{z_0})^2}{q_a^2}(z_0 - z_{0a})^2, \end{aligned} \tag{24}$$

where all derivatives are evaluated at the approximation point, ψ_x collects all second derivative terms with respect to x, with an abuse of notation p_a and q_a are p and q evaluated at the approximation point.

Suppose that log wages are measured with an i.i.d. error. Consider a regression equation of $\log W$ on x, z_0 and t:

$$\log W_i = \beta_0 + \beta_x x_i + \beta_{z_0} z_{0i} + \beta_t t_i + \epsilon_i, \qquad \epsilon_i \sim \text{i.i.d}(0, \sigma_\epsilon^2) \tag{25}$$

We argue that the coefficients in this regression can be given the following interpretation

$$\beta_x = \frac{1}{p_a}\frac{dp}{dx} + \frac{1}{q_a}\left(\frac{\partial q}{\partial x} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial x}\right) \tag{26}$$

$$\beta_{z_0} = \frac{1}{q_a}\frac{\partial q}{\partial z}\frac{\partial z}{\partial z_0} \tag{27}$$

$$\beta_t = \frac{1}{q_a}\frac{\partial q}{\partial z}\frac{\partial z}{\partial t} \tag{28}$$

$$\beta_0 = p_a + q_a - \beta_x x_a - \beta_{z_0} z_{0a} - \beta_t t_a, \tag{29}$$

when the restrictions

$$\text{Cov}[u_a(x, z_0, t), x] = \text{Cov}[u_a(x, z_0, t), z_0] = \text{Cov}[u_a(x, z_0, t), t] = 0 \tag{30}$$

are satisfied, where $u_a(x, z, t)$ is the Taylor approximation error when the approximation point is (x_a, z_{0a}, t_a). In other words, OLS coefficients are consistent and unbiased estimates of polynomial functions of the partial derivatives, evaluated at a point at which the covariances with the regressors vanish. A similar interpretation can be given to a regression involving all quadratic terms. Specifically,

$$\beta_{tt} = \frac{1}{2}\frac{(q_{zz}z_t^2 + q_z z_{tt})q_a - (q_z z_t)^2}{q_a^2}. \tag{31}$$

Under the assumption that $q_{zz} < 0$, $\beta_{tt} < 0$, if $z(t) > z^*/2$, i.e. at least half–way from the limiting value. Therefore, we have the following concluding observations: (1) the Mincer regression will produce a negative quadratic tenure term, provided that in the sample on average $z(t) > z^*/2$ is true; (2) the Mincer regression has an omitted variable problem, unless there is no variation in x in the economy; (3) the Mincer regression should also include the interaction terms, in particular involving skills.