Learning by Problem–Solving

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Abstract

We present a learning by doing model that relates wages and skill development to the level of job complexity. We test our hypotheses about the job complexity effect using German Sample of Integrated Biographies data. We find that when tenure is low, wage growth is positively related to job complexity and negatively related to initial skill level, just as the model would predict. We calibrate the model and find that employees receive a positive net wage premium to the complexity of their job and that professional workers acquire twice as much skills throughout life compared to other high skilled occupations.

1 Introduction

Since the early work of Jacob Mincer on education, experience and earnings, it became a well–known fact that wages increase with work experience at a decreasing rate. Since then, the literature in the field of human capital has accumulated vast evidence about the driving forces behind the wage curves. Wages grow with experience as a result of learning by doing (e.g. Nagypál 2007; Parsons 1972), revealing of the employee to employer match quality (Jovanovic 1979), and acquisition of firm specific (Becker 1962), industry specific (Neal 1995; Parent 2000) and occupation specific (Kambourov and Manovskii 2009) human capital.

What is less known is that wage curves vary widely across occupations. Figure 1 shows occupation–specific hourly wages for different age groups in Germany in 2006. The figure suggests that occupations such as legislators, managers and professionals have wage profiles that are a level shift on the hourly rate scale, and also that the growth rates of wages of these professions are larger and remain positive longer. While ability differences could account for the level effect, they would not explain the observed patterns in the growth rate. It is also difficult to reconcile these differences with a model of uncertainty about the match quality, because there is no compelling reason why match qualities would vary by occupations. Moreover, learning by doing models that relate

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productivity growth to gross output or accumulated investment require either systematic differences in the volume of activities or capital output ratios (in the case of Arrow 1962) across jobs, or sorting by individual differences to explain why learning would happen at a slower or faster rate across different occupations.

In this paper, we put forward a simple model that explains how the differences in task complexity of jobs clustered in different occupations can act as the driving force behind these earnings patterns. We then test the model's implications using both individual–level survey data and the data from the administrative records of the German Social Security.

The basic idea is as follows. We view every job as a collection of tasks ("problems") of varying complexity that need to be accomplished. Every task poses a challenge and at the same time provides an opportunity to learn. Thus, jobs create a learning environment, where employees can learn by solving tasks. Just how much they can learn depends on the gap between their current skill level and the level of task complexity. More complex jobs create a better learning environment, as they provide more complex tasks and in larger quantity. Therefore, occupation–specific wage profiles are possible, because workers in different jobs accumulate skills at different rates.

We find empirical evidence that supports the model's predictions. First, there is a static effect of job complexity. Employers pay wage premia to workers employed in more complex jobs. Second, there is a dynamic effect. Workers in complex jobs develop more skills ceteris paribus.

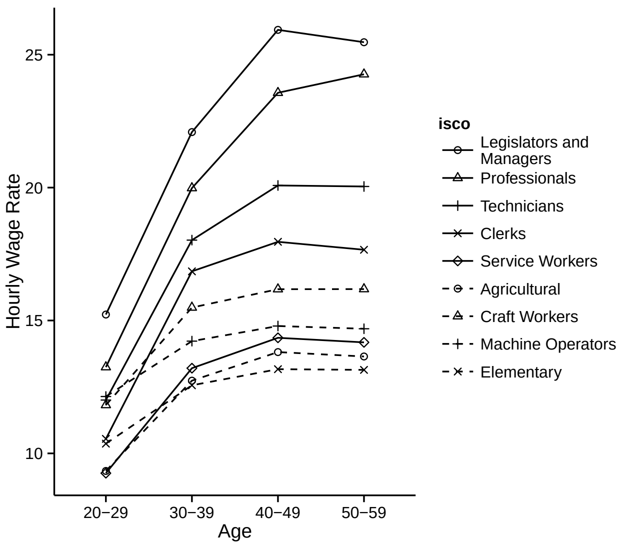

Figure 1: Earnings Profiles by Occupation.

2 The Model

In this section we analyse earnings dynamics in a single good economy. The good can be produced by a variety of jobs with labour as the only factor of production.

Every job consists of a single task, and every task is uniquely characterized by its level of complexity, which we denote by a positive real number x. Higher value of x means a more complex task.

Assume that the market price of a unit of output created by completing a task of complexity x is given by a function $f(x)$, which is positive, monotone and increasing. Such dependency may result from a relation between job complexity and product quality, for example. The dependency of the market price on job complexity creates an immediate wage premium for working in a complex job. This premium is unrelated to the amount of learning or experience accumulated at the job.

Further assume that workers differ in their ability to perform a given task, which is summarized by a task efficiency function $g(x, z)$. The value of g depends on x, since the efficiency of a worker on a given task changes adversely with task complexity, and on skill level z. Since we allow learning, z is a function of tenure, t, and job complexity additionally affects wages through changes in z. We write the wage equation for a worker with skill level $z(t)$ in a job with complexity x as[1]

$$\begin{aligned} &\text{wage} = W(t, x) = f(x)g(x, z(t)), \quad \text{where} \\ &\frac{df}{dx} > 0 \qquad \frac{\partial g}{\partial x} < 0 \qquad \frac{\partial g}{\partial z} > 0. \end{aligned} \tag{1}$$

Let workers increase their skills as time progresses as a result of learning from exposure to a challenging learning environment. Since more complex jobs present a more challenging environment, workers in more complex jobs will accumulate higher skills. We suppose that the growth rate of skills is linearly related to the gap between the level of job complexity and current level of skill:

$$\frac{\dot{z}}{z} = \alpha(x - \eta z) + \beta, \tag{2}$$

where the parameter α characterizes the rate of learning, β is a shift parameter that can be used to model skill depreciation, and η is a normalization parameter.[2] It is assumed that $\alpha > 0, \eta > 0$ and that β is such that the right hand side is not always negative.

Equation (2) may be motivated by figure 2. If we believe that wages reflect skills, then wage growth will also reflect skill growth. We approximate wage growth by a local polynomial regression of log of wage on age, and plot the estimated derivative versus the mean wage in a sample from the SIAB data,

[1]The model allows a probabilistic interpretation in the following sense: working on a task can result in either a success or a failure, which is valued at 0. The likelihood of either outcome depends on a worker's skill level and on task complexity. Output $\tilde{w}$ will be a random variable with mean and variance depending on z. In one possibility, every worker is paid according to actual performance. In the other possibility, workers are paid expected wages

$$W = \mathbb{E}[\tilde{w}|z] = f(x)g(x, z),$$

where g is a bounded function modelling the probability of solving the task. In Jovanovic and Nyarko 1997, the relation is $W = 1 - \sigma_z^2$, where σ_z^2 is the variance of decision error, $0 \leq \sigma_z^2 \leq 1$. σ_z^2 can be written as $1 - g(x, z)$, thus $W = g(x, z)$ and there is no immediate job complexity premium. The dynamics of σ_z^2 is such that the wage–tenure curve is S–shaped (see Jovanovic and Nyarko 1995).

[2]We assume that α is constant. While in general it may depend on such factors as innate learning ability, curiosity etc., it is fixed on the level of an individual.

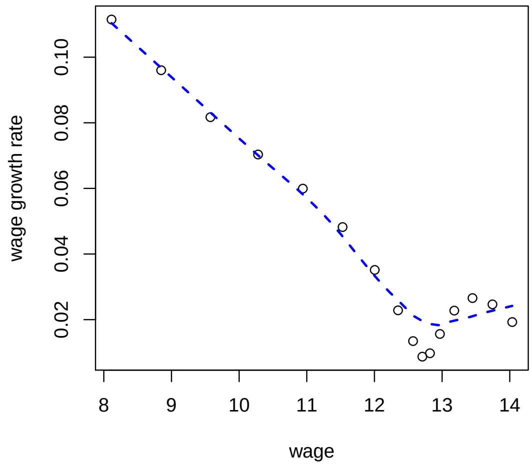

Figure 2: Relation between wage growth rate and wage.

where we restrict the variation in tenure so that the wages are increasing in tenure. The slope of the resulting relation is roughly linear up to high values of wages. We choose a linear relation with a negative slope to capture the essential feature of the data in an analytically tractable way.

From the Bernoulli equation (2) we obtain

$$z(t) = \left[\frac{\alpha\eta}{\alpha x + \beta} + \left(\frac{1}{z_0} - \frac{\alpha\eta}{\alpha x + \beta}\right) e^{-(\alpha x + \beta)t}\right]^{-1}. \tag{3}$$

As $t \to \infty$, $z(t)$ approaches the limit $(\alpha x + \beta)/(\alpha\eta)$ irrespective of the initial value of z. When β is 0, the limiting value is x/η. For either case, we denote it by z^*. In the long run, higher job complexity leads to a higher level of skill.

We can rewrite the solution as

$$z(t) = \left[\frac{1}{z^*} + \left(\frac{1}{z_0} - \frac{1}{z^*}\right) e^{-\alpha\eta z^* t}\right]^{-1}. \tag{4}$$

Since the time derivative of z can be written as

$$\dot{z}(t) = \alpha\eta z(t)[z^* - z(t)], \tag{5}$$

it follows that z grows at a rate proportional to the distance to the limiting value:

$$\frac{\dot{z}}{z}(t) = \alpha\eta[z^* - z(t)]. \tag{6}$$

Figure 3 shows the relations between the level and the rate of growth of wages and job complexity and initial level of skill implied by the model. Panel a) demonstrates that wage profiles of more complex jobs dominate for nearly all values of initial skill. Initial wages may be depressed in a highly complex job, but

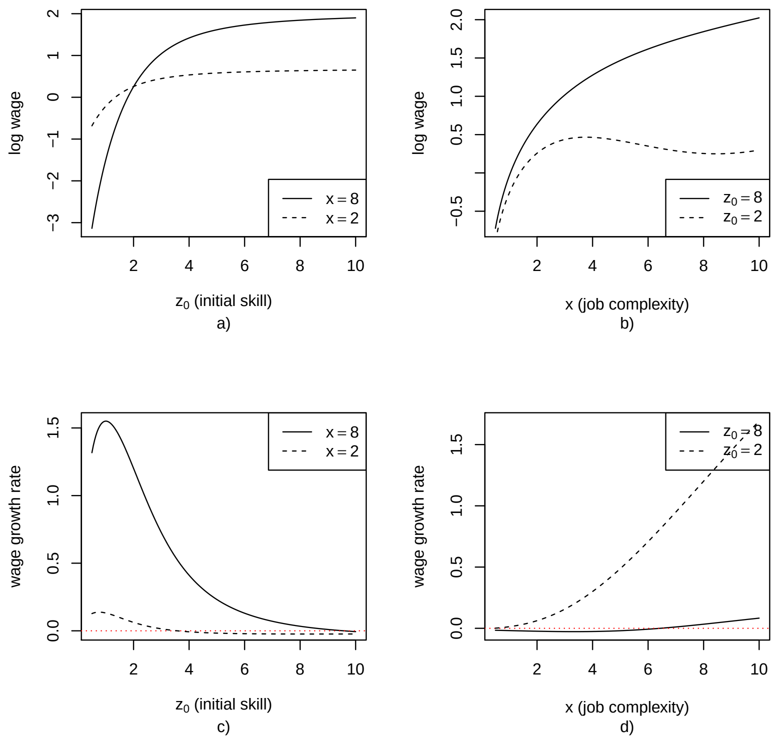

Figure 3: Relation between log wage and initial level of skill and job complexity.

they will eventually catch up and exceed wages in a less complex job. In panel b), workers with relatively higher initial skill earn more in all jobs. In panel c), the rates of growth of wages in a more complex occupation dominate the rates of growth in a less complex one. In the last panel, workers with relatively lower initial skill enjoy higher rates of wage growth for the same occupation. Thus the model is able to explain such facts as higher wages and growth rates of wages in more complex occupations. Additionally, the model predicts that wage growth is generally decreasing in initial skill, conditional on job complexity.

We now turn to analysing properties of wages given the dynamics of z.[3] The elasticities of the wage rate can be expressed as

$$\varepsilon_{wx} = \frac{\partial \log W}{\partial \log x} = \epsilon_{fx} + \epsilon_{gx} + \epsilon_{gz}\epsilon_{zx}, \tag{7}$$

$$\varepsilon_{wz_0} = \frac{\partial \log W}{\partial \log z_0} = \epsilon_{gz}\epsilon_{zz_0}, \tag{8}$$

$$\varepsilon_{wt} = \frac{\partial \log W}{\partial \log t} = \epsilon_{gz}\epsilon_{zt}, \tag{9}$$

where $\epsilon_{fx} = x/f(x)df(x)/dx, \epsilon_{gx} = x/g(x,z)\partial g(x)/\partial x$, etc. We introduce the

[3]We silently assume that $z < z^*$. Since wages are typically growing in tenure and not falling, this seems to be the dominant case.

following assumptions:

A1. $\epsilon_{fx}, \epsilon_{gx}$ ϵ_{gz} are bounded functions of x and z.

A2. $\epsilon_{fx}, \epsilon_{gx}$ ϵ_{gz} are constant functions of x and z.

While we mostly invoke A2, some statements can be derived with A1 alone. From

$$\epsilon_{zx} = zx\left[\frac{1}{\eta z^{*2}}\left(1 - e^{-\alpha\eta z^* t}\right) + \alpha t e^{-\alpha\eta z^* t}\left(\frac{1}{z_0} - \frac{1}{z^*}\right)\right], \tag{10}$$

$$\epsilon_{zz_0} = \frac{z}{z_0}e^{-\alpha\eta z^* t}, \tag{11}$$

$$\epsilon_{zt} = \alpha\eta(z^* - z)t, \tag{12}$$

it follows that

$$\lim_{t\to 0}\epsilon_{zx} = 0 \qquad \lim_{t\to 0}\epsilon_{zz_0} = 1 \qquad \lim_{t\to 0}\epsilon_{zt} = 0 \tag{13}$$

$$\lim_{t\to \infty}\epsilon_{zx} = 1 - \frac{\beta}{\alpha x + \beta} \qquad \lim_{t\to \infty}\epsilon_{zz_0} = 0 \qquad \lim_{t\to \infty}\epsilon_{zt} = 0 \tag{14}$$

Clearly, $\epsilon_{zx} > 0, \epsilon_{zz_0} > 0$ and $0 < \epsilon_{zt} < z^*$ for any $t > 0$. Using these properties we can find $\epsilon_{fx} + \epsilon_{gx}$ and ϵ_{gz} from $\varepsilon_{wx}, \varepsilon_{wz_0}$ and ε_{wt} using assumption A2. First, we can determine ϵ_{gz}, since it is equal to $\lim_{t\to 0}\varepsilon_{wz_0}$. Then we can determine $\epsilon_{fx} + \epsilon_{gx}$ from $\lim_{t\to 0}\varepsilon_{wx}$.

Table 1: Estimates of elasticities for tenure groups.

tenure	ε_{wx}	ε_{wz0}	ε_{wt}	ϵ_{zz_0}	ϵ_{zt}	ϵ_{zx}
(0,5]	0.21	0.89	0.18	1.00	0.20	0.00
(5,10]	0.29	0.88	0.25	0.98	0.28	0.08
(10,15]	0.32	0.79	0.35	0.88	0.40	0.11
(15,20]	0.35	0.72	0.48	0.80	0.54	0.14
(20,25]	0.38	0.62	0.59	0.69	0.66	0.17
(25,30]	0.41	0.57	0.60	0.63	0.67	0.20

Table 1 presents estimates of $\varepsilon_{wx}, \varepsilon_{wz_0}, \varepsilon_{wt}$, which are broadly consistent with our a priori expectations. We find that the job complexity elasticity of wage is increasing over time, just as would be predicted from ϵ_{zx} eventually reaching higher values. The elasticity with respect to z_0 is decreasing, and with respect to t increasing and reaching a peak, though we don't observe the latter elasticity eventually decreasing.[4]

Under the assumptions that the initial values of z are mostly below the steady–state, we summarize the properties of wages in

Proposition 1. a) *for any x, there is a t^* such that $W(t)$ is concave for all $t > t^*$ and convex otherwise;*

[4]This is consistent with the Mincer model results, where the tenure elasticity is first increasing, then decreasing. However, the elasticity with respect to z_0 is time invariant in the Mincer specification. One could argue that the observed pattern is due to depreciation of skills acquired at school. This would imply that the returns to schooling would be decreasing in age. We find that returns to schooling follow a U shape in the data.

b) *the growth rate of wages is increasing in x and decreasing in z at t=0.*

Proof.

$$\begin{aligned}
\frac{\partial^2 W}{\partial t^2} &= f(x)\left[g_{zz}\dot{z}^2 + g_z\ddot{z}\right] = f(x)\dot{z}\left[g_{zz}\dot{z} + g_z\alpha\eta(z^* - 2z)\right] \\
&= f(x)\dot{z}\left[\frac{g_z(\epsilon_{gz} - 1)}{z}\dot{z} + g_z\alpha\eta(z^* - 2z)\right] \\
&= \{\alpha\eta f(x)\dot{z}g_z\}\left[(z^* - z)\epsilon_{gz} - z\right].
\end{aligned} \tag{15}$$

The term inside the left brackets is always positive and approaches 0 as $t \to \infty$. The term on the right is negative for all $t > t^*$ and positive for $t < t^*$, where

$$t^* = -\frac{1}{\alpha\eta z^*}\log\frac{z_0}{\epsilon_{gz}(z^* - z_0)}. \tag{16}$$

Thus the wage curve has a convex segment followed by a concave one.

To prove *b)*,

$$\lim_{t\to 0}\frac{\dot{W}}{W} = \lim_{t\to 0}\epsilon_{gz}\frac{\epsilon_{zt}}{t} = \epsilon_{gz}\left[\beta + \alpha x - \alpha\eta z_0\right]. \tag{17}$$

□

We also examine conditions for wages to be increasing in job complexity.

$$\frac{\partial W}{\partial x} = f_x g + f[g_x + g_z z_x] = \frac{W}{x}\left[\epsilon_{fx} + \epsilon_{gx} + \epsilon_{gz}\epsilon_{zx}\right] \tag{18}$$

Thus it is sufficient for $\partial W/\partial x > 0$ that $\epsilon_{fx} + \epsilon_{gx} > 0$. In the long run we have

$$\lim_{t\to\infty}\frac{\partial W}{\partial x} = \frac{W(x, z^*)}{x}\left[\epsilon_{fx} + \epsilon_{gx} + \epsilon_{gz}\left(1 - \frac{\beta}{\alpha x + \beta}\right)\right] \tag{19}$$

and therefore if $\alpha x + \beta > 0$, then

$$x > -\frac{\beta}{\alpha}\frac{\epsilon_{fx} + \epsilon_{gx}}{\epsilon_{fx} + \epsilon_{gx} + \epsilon_{gz}} \tag{20}$$

is the necessary and sufficient condition for long–term wages to be increasing in x.

3 Data, Samples and Measurement Issues

Our data come from two sources. The first source is the BIBB/IAB and BIBB/BAuA Surveys on Qualifications and Working Conditions in Germany (BIBB Surveys). The second source is the administrative records of the German Social Security System as prepared by the Institute for Employment Research (IAB).

3.1 BIBB Surveys

The BIBB Surveys are cross–sectional surveys of the German working population, which investigate job tasks, technologies, organizational and technological changes, skills and knowledge, as well as the working conditions of German employees (Rohrbach-Schmidt 2009). The survey has been repeated seven times since 1979. For the purpose of our research we are using two latest waves: 2005/2006 and 2011/2012. Each of these two waves surveys about 20,000 German employees. For each of these employees we have an extensive description of the tasks performed at the job and their qualifications including general education and months of vocational training. The BIBB Surveys are used for constructing measures of job complexity.

3.2 German Social Security Records

The IAB Employment History (BeH) contains the employment history of all Germans subject to social security since 1975 for West and since 1992 for East Germany, which covers about 80 percent of the German labour force. Civil servants and self–employed are not included in the dataset. The BeH contains information about education, occupation, industry, and work location of people on the 30th of June each year. From the available information we can calculate age, general experience, firm and occupational experience of workers. Over 350 occupations[5] are available in the data, but the IAB recommends grouping these in about 120 occupational groups to avoid false precision.

We restrict the sample to individuals who had at least seven consequent years of work experience in the same occupation. This ensures that there are no strong attrition effects in the data. From this group, we draw a random sample of 25,000 individuals to be used in a cross–section regression of wage growth based on equation (17) obtained in proposition 1. We measure wage growth rate as the average of yearly growth rates of wages in the first four observed years. We use occupation experience as a measure of t and years of schooling obtained from the highest qualification level as a proxy measure of $z(0)$. We assign a value of x to each individual observation based on the occupational code, thus all individuals who have the same KldB88 occupational code have the same value of x. We explain the specifics of obtaining our measure of x in the next subsection.

3.3 Job Complexity

The BIBB Surveys ask the following eights questions that we think are related to the level of job complexity:

> How often does it happen at your work that you:
>
> - have to react to unexpected problems and resolve these?
> - collect, investigate and document data?
> - have to make difficult decisions independently and without instructions?
> - have to recognize and close own knowledge gaps?

[5]This is the 3–digit German Classification of Occupations 1988 (KldB88)

Table 2: Factor loadings after Principal Component Analysis.

	2006	2012
information	.570	.571
newproblems	.645	.619
difficultdecisionsalone	.598	.597
knowledgegaps	.546	.490
newtasksthink	.595	.585
processimprove newideas	.578	.549
multitask	.520	.511

- are faced with new tasks which you first have to understand and become acquainted with?
- have to improve processes or try out something new?
- have to keep an eye on many different processes at the same time?

We apply principal component analysis (PCA) for extracting common variance within a set of correlated variables that we believe are strongly related to job complexity. In our case PCA is particularly appropriate not only because it can reduce the dimensionality of the eight questions, but also help us examine whether there is indeed a single common factor that is associated with job complexity.

We conduct PCA separately for each wave. It turns out that all eight questions load very high on the first factor, both in 2006 and in 2012. In both analyses there is also a second factor in which however the factor loadings are very poor. While the first factor has eigenvalue of 2.35 (2.21 in 2012), the second has an eigenvalue of only 0.06 (0.10 in 2012). Therefore, the eigenvalues suggest that we should only keep the first factor in each wave. What is more, the factor loadings in the two independent analyses are remarkably similar. Table 2 shows the factor loadings on each of the eight questions in the PCA. As a rule of thumb loadings above 0.5 are considered high. We call this factor job complexity, because we believe that it captures the intensity of problem–solving at the job.

Figure 4 compares the factor distributions. Both distributions are left–skewed, meaning that the mass of the workers in Germany report high incidence of new problem arrival at their jobs. The t–test of the distributions (t=-0.000) shows that their means are statistically the same.

4 Calibration

We calibrate the model to examine its plausibility and obtain estimates of its parameters. We use (17) to estimate α and η from the coefficients of job complexity and $z(0)$ in a cross sectional regression with wage growth as the dependent variable. The fit is summarized in table 3. Due to data limitations we have to use education as a crude proxy for $z(0)$, which will bias the coefficient of $z(0)$ upward. We find evidence of a positive effect of job complexity on wage growth and a negative effect of initial skill.

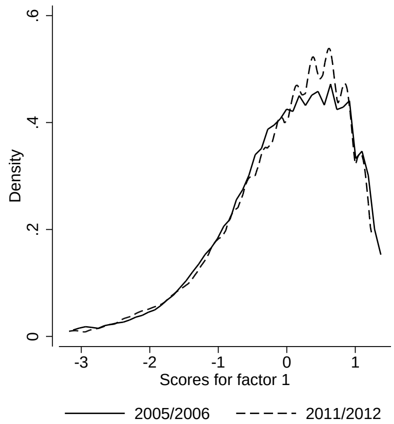

Figure 4: Distributions of job complexity factor.

Model 1 is a baseline model which does not include job complexity. Model 2 is our equation (17) with additional controls for gender and West Germany dummy. Model 3 adds controls for year, birth cohort and industry. We note that the rate of wage growth is notoriously hard to predict. Overall, the results of estimation of Model 2 support the implications of the model and are robust to including several controls.

From the earlier results we established an estimate of $\hat{\epsilon}_{gx} = 0.89$. The regression of the rate of wage growth yields an estimate $\hat{\alpha} = 0.07$. We get an estimate of η from the ratio of the coefficients of school and x. In order to calibrate β, we use

$$\frac{\epsilon_{zt}}{\epsilon_{zz_0}} = \alpha\eta(z^* - z_0)t. \tag{21}$$

The graph of $\epsilon_{zt}/\epsilon_{zz_0}$ is approximately linear in age, and the estimated slope is $\hat{s}_t = 0.037$ (s.e. 0.003).

$$\hat{\beta} = \hat{\alpha}\hat{\eta}(\hat{s}_t + z_0) - \hat{\alpha}x. \tag{22}$$

We evaluate this expression at sample average values $\bar{x} = 0.41$ and $\bar{z}_0 = 12.57$ and obtain $\hat{\beta} = 0.015$. The estimates are collected in table 4.

Figure 5 plots calibrated skill profiles for different occupations where the starting value of z_0 is taken to be the average within the corresponding occupation group. More complex occupations have higher values of starting level of skill as measured by years of schooling (the correlation coefficient of school and x in the entire SIAB sample is 0.57). In addition to that, different occupations accrue unequal amount of skill: while professionals on average gain skills equivalent to about two school years after 30 years of work, service workers gain less than one schooling year equivalent. In fact, clerks and machine operators even lose skills on average for the same reference period.

Table 3: Estimation results for wage growth equation.

	Model 1	Model 2	Model 3
(Intercept)	0.114^{***}	0.118^{***}	0.126^{***}
	(0.000)	(0.000)	(0.000)
femaleTRUE	-0.001	0.001	0.002
	(0.000)	(0.000)	(0.000)
westTRUE	-0.003	-0.004	-0.009
	(0.000)	(0.000)	(0.000)
school	0.000	-0.003^{***}	-0.003^{**}
	(0.000)	(0.000)	(0.000)
x		0.062^{***}	0.069^{***}
		(0.000)	(0.000)
R^2	0.000	0.001	0.049
Adj. R^2	0.000	0.001	0.046
Num. obs.	23340	23340	21050

Robust errors. Model 3 also includes industry, year and cohort dummies.

Table 4: Calibrated parameters.

parameter	$\hat{\epsilon}_{gz}$	$\widehat{\epsilon_{fx} + \epsilon_{gx}}$	$\hat{\alpha}$	$\hat{\eta}$	$\hat{\beta}$
estimate	0.89	0.21	0.07	0.05	0.02

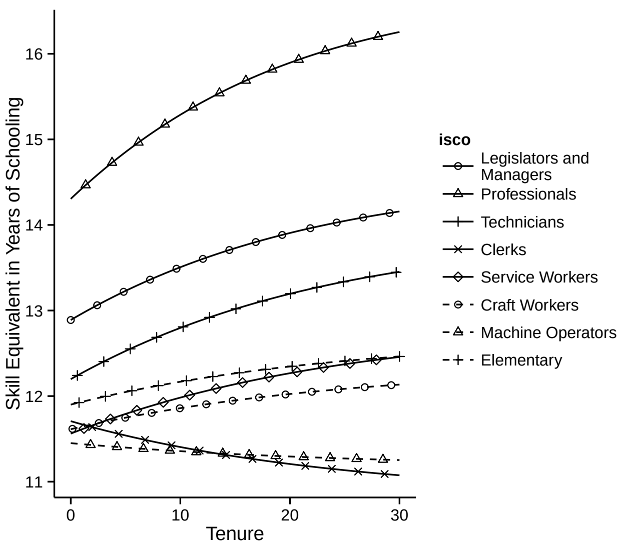

Figure 5: Calibrated skill profiles.

5 Conclusion

We demonstrated that the existence of occupation–specific Mincer profiles of earnings may be explained by differences in job complexity and learning induced by exposure to problem solving in more complex jobs. Jobs with higher complexity have higher static wage premia. Additionally, working in a complex jobs increases the level of skill, even after controlling for education differences.

One empirical fact that we are not able to explain at the moment is the absence of a decline in variance of earnings with tenure. In our model, the level of skills of every worker eventually converges to the steady state value z^*. Thus, the unconditional variance of wages shrinks over time. We think that labour market information asymmetries may be responsible for much of variance dynamics that can not be explained by the model.

A question that remains unanswered at this point is whether skills acquired by learning from more complex jobs are transferable to other jobs. If they are, there might be scope for designing policies that encourage workers to take challenging jobs and encourage employers to challenge their workers at the job. If these skills are not transferable, then one implication is that quit rates would be lower in more complex jobs ceteris paribus. Moreover, more work should be done to understand the matching of workers with certain abilities to jobs of certain complexity in order to understand the scope between ability and job complexity where learning can take place.

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A Obtaining the Mincer Model

We now show how the standard Mincer model can be obtained as an arbitrary approximation to our model. We consider z to be a function of x, z_0 and t. We assume f and g have continuous second order derivatives. Let the point of approximation be (x_a, z_{0a}, t_a). Then take the log of the wage equation and calculate the Taylor polynomials,

$$\log W = \log f(x) + \log g(x, z(x, z_0, t)) \tag{23}$$

$$\begin{aligned} T_1 \log W = {} & \log f_a + \log g_a + \left[\frac{1}{f_a}\frac{df}{dx} + \frac{1}{g_a}\left(\frac{\partial g}{\partial x} + \frac{\partial g}{\partial z}\frac{\partial z}{\partial x}\right)\right](x - x_a) \\ & + \frac{1}{g_a}\frac{\partial g}{\partial z}\left[\frac{\partial z}{\partial z_0}(z_0 - z_{0a}) + \frac{\partial z}{\partial t}(t - t_a)\right], \end{aligned} \tag{24}$$

$$\begin{aligned} T_2 \log W = {} & T_1 \log W + \psi_x (x - x_a)^2 \\ & + \frac{1}{2}\frac{(g_{zz} z_t^2 + g_z z_{tt}) g_a - (g_z z_t)^2}{g_a^2}(t - t_a)^2 \\ & + \frac{1}{2}\frac{(g_{zz} z_{z_0}^2 + g_z z_{z_0 z_0}) g_a - (g_z z_{z_0})^2}{g_a^2}(z_0 - z_{0a})^2, \end{aligned} \tag{25}$$

where all derivatives are evaluated at the approximation point, ψ_x collects all second derivative terms with respect to x, with an abuse of notation f_a and g_a are f and g evaluated at the approximation point.

Suppose that log wages are measured with an i.i.d. error. Consider a regression equation of $\log W$ on x, z_0 and t:

$$\log W_i = \beta_0 + \beta_x x_i + \beta_{z_0} z_{0i} + \beta_t t_i + \epsilon_i, \qquad \epsilon_i \sim \text{i.i.d}(0, \sigma_\epsilon^2) \tag{26}$$

We argue that the coefficients in this regression can be given the following interpretation

$$\beta_x = \frac{1}{f_a}\frac{df}{dx} + \frac{1}{g_a}\left(\frac{\partial g}{\partial x} + \frac{\partial g}{\partial z}\frac{\partial z}{\partial x}\right) \tag{27}$$

$$\beta_{z_0} = \frac{1}{g_a}\frac{\partial g}{\partial z}\frac{\partial z}{\partial z_0} \tag{28}$$

$$\beta_t = \frac{1}{g_a}\frac{\partial g}{\partial z}\frac{\partial z}{\partial t} \tag{29}$$

$$\beta_0 = f_a + g_a - \beta_x x_a - \beta_{z_0} z_{0a} - \beta_t t_a, \tag{30}$$

when the restrictions

$$\text{Cov}[u_a(x, z_0, t), x] = \text{Cov}[u_a(x, z_0, t), z_0] = \text{Cov}[u_a(x, z_0, t), t] = 0 \tag{31}$$

are satisfied, where $u_a(x, z, t)$ is the Taylor approximation error when the approximation point is (x_a, z_{0a}, t_a). In other words, OLS coefficients are consistent and unbiased estimates of polynomial functions of the partial derivatives, evaluated at a point at which the covariances with the regressors vanish. A similar interpretation can be given to a regression involving all quadratic terms. Specifically,

$$\beta_{tt} = \frac{1}{2} \frac{(g_{zz} z_t^2 + g_z z_{tt}) g_a - (g_z z_t)^2}{g_a^2}. \tag{32}$$

Under the assumption that $g_{zz} < 0$, $\beta_{tt} < 0$, if $z(t) > z^*/2$, i.e. at least half–way from the limiting value. Therefore, we have the following concluding observations: (1) the Mincer regression will produce a negative quadratic tenure term, provided that in the sample on average $z(t) > z^*/2$ is true; (2) the Mincer regression is misspecified due to the omission of the terms involving x, unless there is no variation in x in the economy; (3) the Mincer regression should also include the interaction terms, in particular involving skills.